Appendix

The Wealth concentration and Pikettisation model: Explanation from static movie frames

We provide below detailed explanations of selected film frames, as referenced by ((1)) . . . ((6)) in the main document.

((1)) Fig.1 Movie inequality_N12_T2000_r89_E at t=365 days (frame 40)

The population is N=12 individuals, the movie goes to day 2000, we stop at t=350 to learn how to read the four recurrent graphs that we use in all movies. Fig.2 is dedicated to nearly the same exercise, but for the N=3600 case, for the sake of completeness.

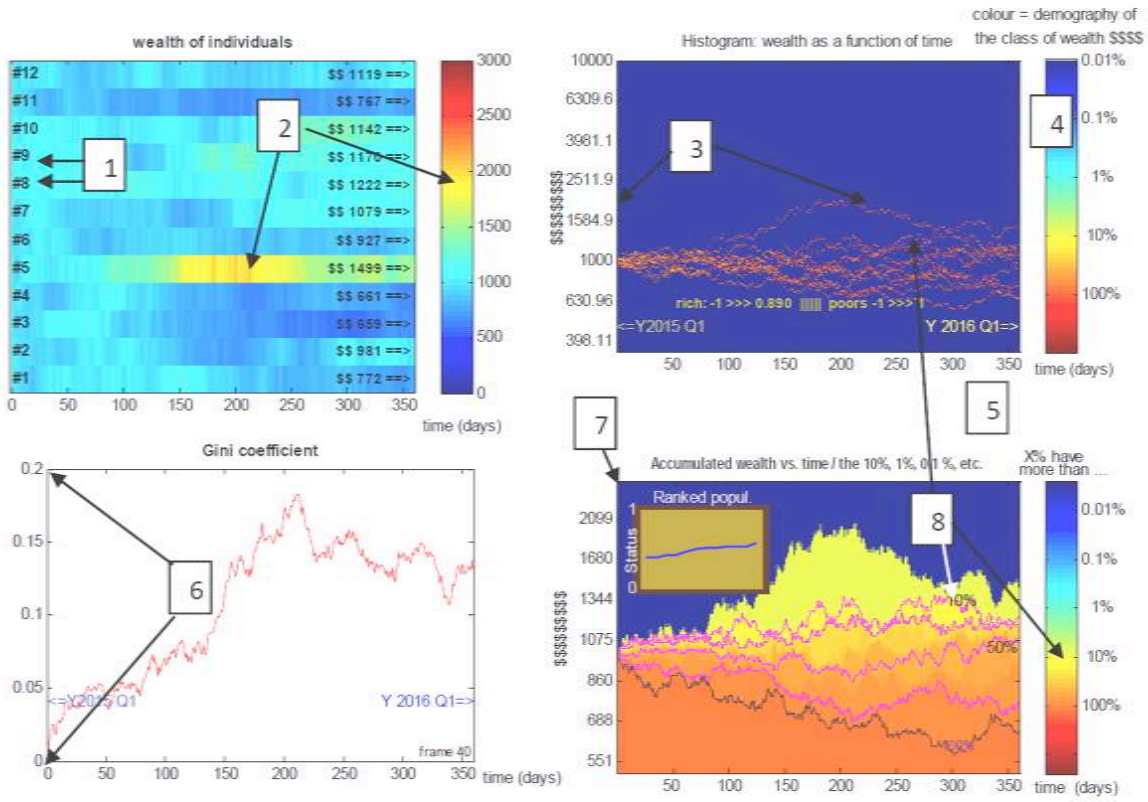

Fig. 1 Caption:

1: Each of the 12 lines of this graph tracks a particular individual, we point the 8-th and 9-th for instance.

2: We use a color map to represent wealth K_i, with the legend ('colorbar') on the right, in a fixed range 0 to 3000. Agent #5 reaches about $$2000 around day 200.

3: Histogram of wealth. This will be more useful in later movies with higher N. For the time being, the histogram is so sparse with 12 of its 300 bins at nonzero value that it resembles a simple plot of the wealth trajectory. The wealthy #5 at time 200 is the outlier trajectory indicated, at about $$2000.

4: The color look-up table will be useful for denser histograms, see ((2)).

5: The horizontal time scale is in days. To make the conversion to years convenient, we assume T0=Y2015, Q1, and thus here, at T=365, we are entering Y2016, Q1.

6: Gini coefficient. I use the standard definition.

$$G = \frac{2\Sigma_{i=1}^{n} i y_i}{n\Sigma_{i=1}^{n} y_i} - \frac{n+1}{n}$$

Note that the scale is automatically adjusted along the y axis, causing the curve to change aspect as time passes.

7: Cumulated histogram showing percentiles. The black line corresponds to 100%, the wealth of the poorest. The four violet lines correspond to the 80%, 50%; 20%, 10% richest. Other lines will appear in Fig.2. Here, one individual represents at least 1/12=8.3% of the total.

8: We see that the 10% line is drawn at the second wealth value, as one goes there from 2/12 to 1/12, or 16.6 to 8.3 %. These awkward considerations are an artifact of the small population, they become more natural for a more continuous distribution. The larger population will also allow us to draw percentiles frequently mentioned in Piketty's remarks, namely 3%, 1%, 0.3%, 0.1%.

((2)) Fig.2 Movie inequality_N3600_T35000_r100_E at t=900 days (frame 26).

There is a much larger N=3600 population. Histograms are no longer "sparse", revealing in later figures the role of the wealthiest.

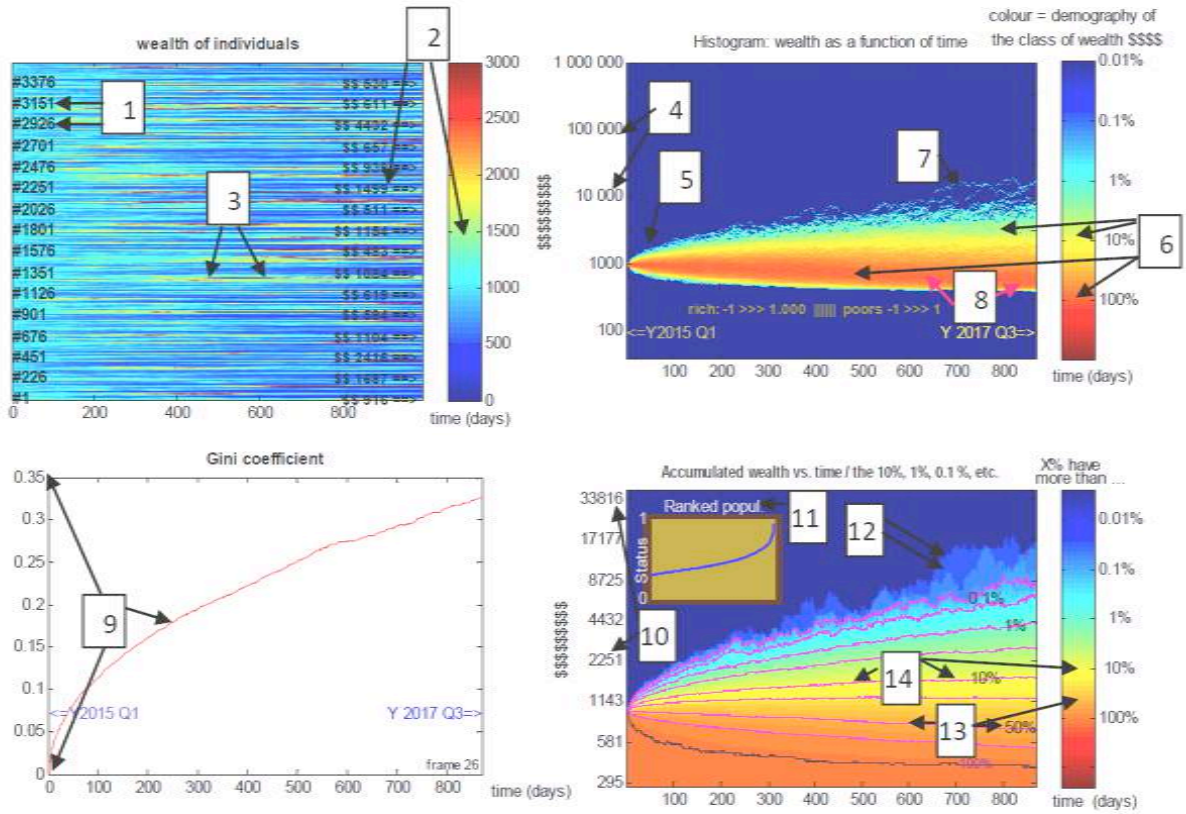

Fig. 2 Caption:

1: Each of the many lines of this graph tracks a particular individual, the i-th one, we indicate #i.

2: We use a color map to represent wealth K_i, with the legend ('colorbar') on the right, in a fixed range 0 to 3000.

3: The typical duration of a "rich period" (a yellowish or reddish 'scar' here) is neither long nor short in this random walk model with a fixed fraction "bet". However, the poor do not get out quick from low levels, because it takes time when betting small amounts. Some blue trajectories last longer. A zoom and a full statistical study would be needed to assess this more rigorously.

4: Histogram of wealth. Note the logarithmic scale on the left. We use 300 “bins” (slices of wealth) to cover four decades. Typical threshold values include [561.5 580.8 600.9, etc.] in the “low wealth” range and [41 435 42 863 44 339, etc.] in the “high wealth” range.

5: At the beginning, we have a spread whose typical width grows with square root of time (parabolic shape), as typical for random walks.

6: The colour representation here is that the majority slice is “100%” and is red, whatever its size. Its wealth goes down from 1000 to 800 or 700 in the period shown: any inequality lowers the median/majority from the mean, here forced at 1000.

7: The happy few are those bluish dots swinging at the top. Not yet millionaires here, but having 20 times the minimum level.

8: A large middle class lies somewhat above the poverty threshold set here at K=400. Inevitably, a fraction shall slowly tend to “land” at K=400…

9: Gini coefficient. I use the standard definition.

$$G = \frac{2\sum_{i=1}^{n} i y_i}{n\sum_{i=1}^{n} y_i} - \frac{n+1}{n}$$

Note that the scale adjusts automatically along the y axis, causing the curve to change aspect as time passes.

10: In this graph, I use log scale along y again, but within the bounds reached by the current wealth income. This makes the initial spread more visible, and compresses vertically the scale only when needed.

11: This small plot gives the distribution of the status as defined by $S_i=K_i/(\text{mean}(K)+K_i)$. There are a few tricks, e.g. sampling the extreme i finely, not the medium i values, to make this figure decent, but they do not change the result. The lower status (left side) is typically tending toward 0.286=Kp/(1000+Kp) with Kp=400. The upper status (right side) tends to one if for the corresponding i we have $K_i >> 1000$.

12: In blue shades, the highest wealths that appeared as dots in the above graph appear here as the upper percentiles. It is seen that these outliers are often well above their immediate followers.

13: The median of the distribution, (50% earn less, 50% earn more) corresponds to a clear orange shade. The different lines refer to 100% (lower bound=poorest wealth, black line), 80%, 50%; 20%, 10%, 3%, 1%, 0.3%, 0.1% (in this case the 3 wealthiest out of 3600 people).

14: Between 50% and 10% (yellow-green color) lies what would be called the “upper middle class”.

((3)) Fig.3: Movie inequality_N3600_T35000_r100_E at t=6300 days (frame 46).

By getting to this longer time, we have let the highest wealth amounts increase. We will see how they induce “FT fits”, looking here at events in the triggering period.

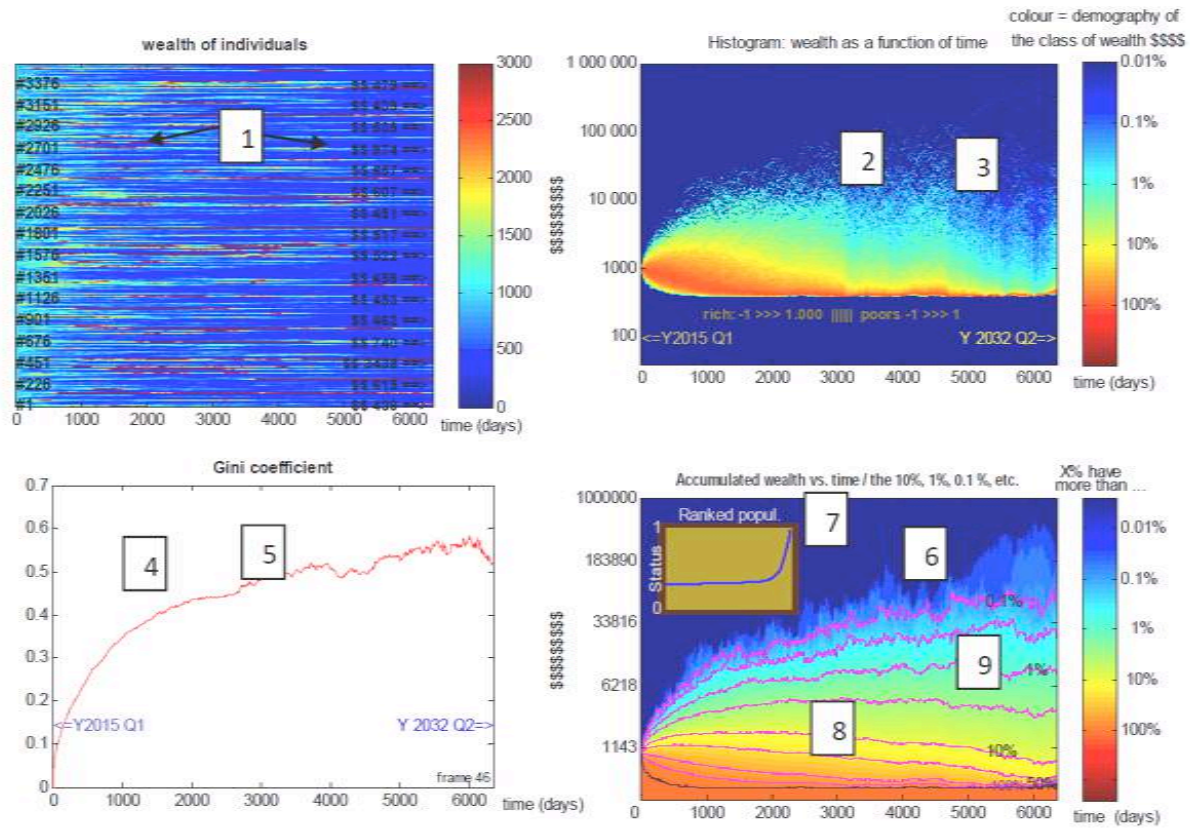

Fig.3 Caption:

1: As time goes to 6300 days, the average color gets bluer as the majority of the population goes to the bottom of the distribution.

2: As the process of making people poorer goes on, it reaches a point where the bottom slice, that just above K=Kp, gathers more population, being the “ground” where anybody getting poorer lands. This can happen somewhat suddenly, because given the fluctuations of the richest, the process starts to be assisted by the forced averaging effect (at $$1000), pushing suddenly by, say, minus 3-4% typically the second slice (the range $$414-428) into the ground slice (the range $$400-414). Once this population slice is larger than all the others, it causes the relative fraction of all slices lying above to diminish, thus a “bluishment” of the colors. The individuals in all the slices above the bottom ones, however, are not undergoing more than a couple of percent downward.

3: The process described above is repeated at t~4600 days when further shocks are inflicted by the richest elements’ fluctuations. These signs are precursors of the “FT fit” but they do not yet shake the whole society, they “only” mark the formation of a large proletarian population.

4: The Gini coefficient started growing as before (parabolic shape typical of random walk) but now quickly exceeds 0.5

5: At that point, the Gini coefficient is affected by the rather abrupt changes in the distribution induced by super-rich spreading their influence to the whole system through the averaging, and it becomes noisy.

6: We now see large inequalities with wealth well above $$100 000 occurring around t=6000 days.

7: The status distribution shows that only a few people have S close to 1, the vast majority having low status.

8: We see that the 10% class possesses decreasing wealth from t=2000 days on.

9: Only the 1% still look stable, though with growing fluctuations. Overall we are at the verge of the system's first serious "FT fit".

((4)) Fig.4: Movie inequality_N3600_T35000_r100_E at t=17400 days (frame 62)

We will now see a severe "FT fit".

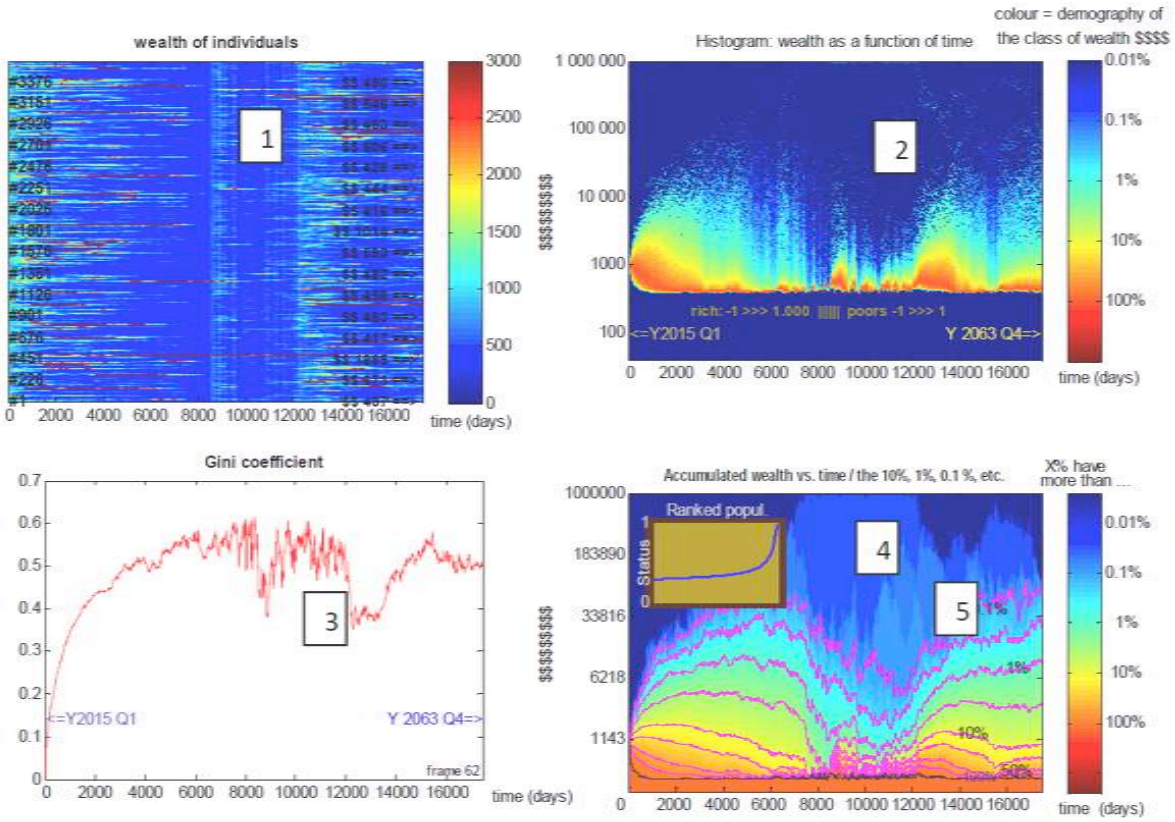

Fig. 4 Caption:

1: The longer story shows some recoveries happening after shocks, most clearly here at t=12000 days.

2: Around t=12000 days, the fluctuation has been large enough to allow many poor people to restart their social climb for a few thousand days, at the apparent expense of very few very rich persons, as we shall see.

3: The Gini coefficient has a lot of "noise" after 6000 days. It is shaken when going through the "FT fit" event, but if you look carefully to the sudden drop and the sustained "trough" at Gini=0.4 around t=12000 to 13000 days, it corresponds to a period where most super large wealth have eventually collapsed, in a redistributive manner in the sense explained in the remark above.

4: The "FT fit" due to huge concentration of wealth (over $$1 000 000) in a couple of hands is clear during the period t=8000-12000 days.

5: However, after the fit and recovery, the system again evolves much as it did around=4000-6000 days before the FT fit with the 10% steadily losing their shares around 14 000 days. The large fluctuations of the several hyper rich that have survived can still cause some small fits, at t=15500 for instance.

((5)) Fig. 5: Movie inequality_N3600_T35000_r098_E at t=20500 days (frame 65)

We use cPik=0.01 for the "Piketty-inspired" coefficient that biases the wealth concentration mechanism. The rich now have a slightly lower probability of gain than the poorest: the upper limit is 0.98=1-2*cPik for status S=1, and 0.9943 for those at K=Kp given their lower S=Sp=0.286.

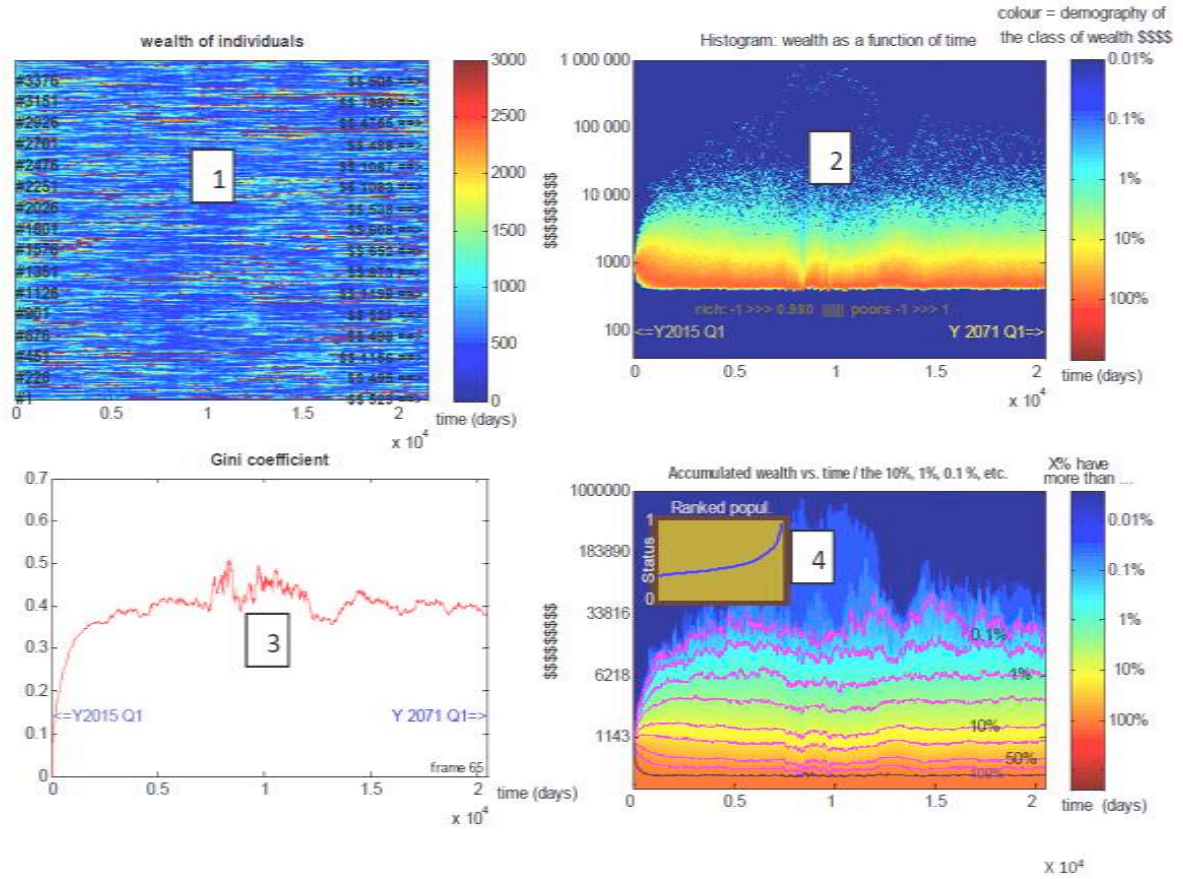

Fig.5 Caption:

1: The overall landscape looks almost quiet, there is no salient feature.

2: The random walk turned into an almost stable system. In this graph, we see a "FT fit precursor" around t=8000 days (indicated as 0.8 10^4), in the form of a narrow dip. But the system now recovers almost immediately and shows "gentle" fluctuations.

3: The "FT fit precursor event" causes a sizable perturbation in the Gini coefficient.

4: As we can now see, a single wealth approaching 1 000 000 has emerged in the period around 8000 causing eventually the perturbations (such large wealth have a potential of exchange of 60 000 "euros" a day, therefore, in our model, over 100 times the wealth of a poor). Still, this large wealth survives for about a decade (3000 days) and its fall, around t=12000, gives a small kick up to everybody, but again, no strong fit. Note that the status curve now has a gentle slope throughout its low-part (left side) distribution, which means that the proletarian class is a continuum, and social climb is not a dream.

((6)) Fig.6 Movie inequality_N3600_T35000_r085_E at t=19500 days (frame 64)

The richest now have a quite smaller probability of gain than the poorest: 0.85 instead of about 0.96 (cPik=0.075).

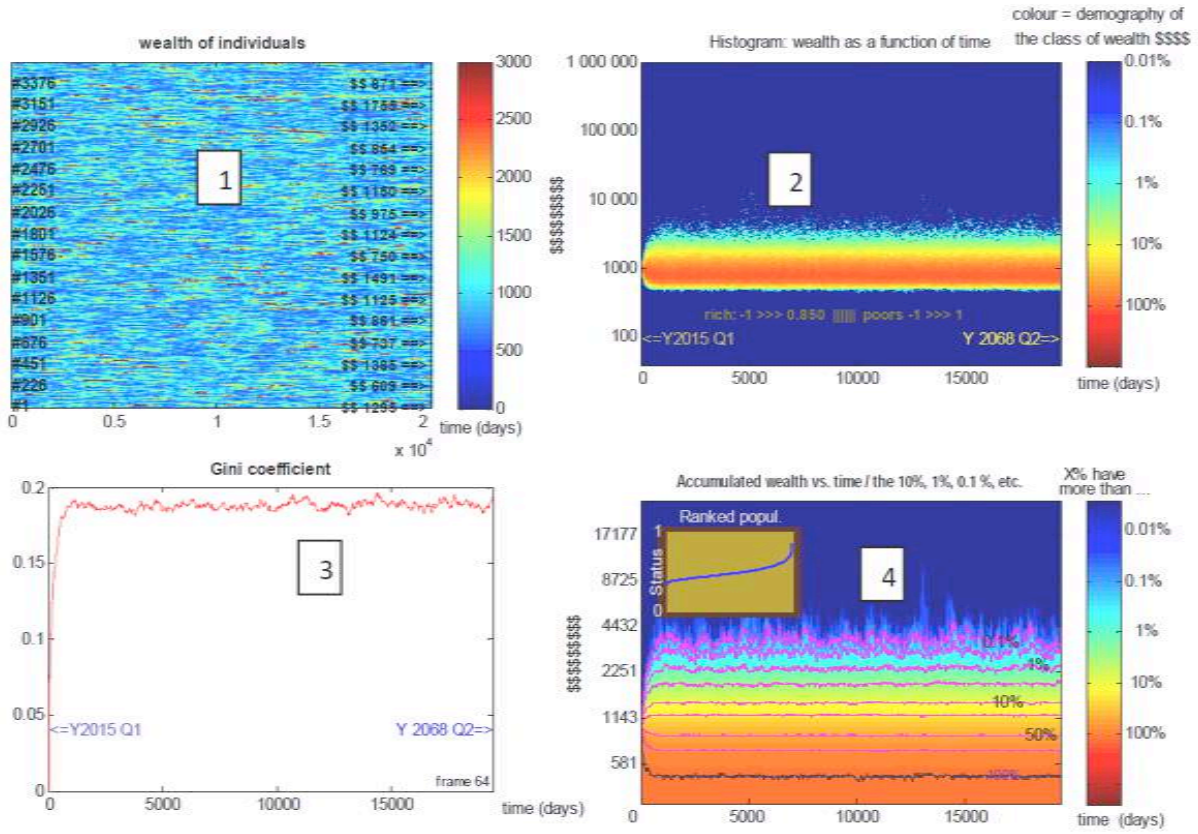

Fig.6 Caption:

1: The overall landscape is quiet, with smaller “speckles” than before, meaning that fluctuations are also of shorter extent in time.

2: The random walk is strongly stabilised (before t=1000 days) and devoid of detectable collective fluctuations.

3: The Gini coefficient is now stabilised around 0.19 with minor jitter around its average.

4: The distribution when stabilised now has a median value around 800-900. Jitter gently affects the 1% income line through time, and a more visible jitter the 0.1% line, but we see that the largest wealth is confined between 4000 and occasionally 10 000, 25 times the minimum wealth. As for the status, its final upward “hockey stick” rise is smaller in extent than those of all previous cases. The wealthiest are in small number, and not enormously richer than the middle or upper-middle class.